ISSN 1392-8619 print/ISSN 1822-3613 online
ÛKIO TECHNOLOGINIS IR EKONOMINIS VYSTYMAS
TECHNOLOGICAL AND ECONOMIC DEVELOPMENT OF ECONOMY
http://www.tede.vgtu.lt
2006, Vol XII, No 1, 56–61

HIGH ORDER ITERATIVE METHODS FOR DECOMPOSITION-COORDINATION PROBLEMS

Otu Vaarmann

*Tallinn University of Technology, Institute of Cybernetics,
Akadeemia tee 21, 12618 Tallinn, Estonia
E-mail: vaarmann@edu.ttu.ee*

Received 3 October 2005; accepted 4 January 2006

Abstract. Many real-life optimization problems are of the multiobjective type and highdimensional. Possibilities for solving large scale optimization problems on a computer network or multiprocessor computer using a multi-level approach are studied. The paper treats numerical methods in which procedural and rounding errors are unavoidable, for example, those arising in mathematical modelling and simulation. For the solution of involving decomposition-coordination problems some rapidly convergent interative methods are developed based on the classical cubically convergent method of tangent hyperbolas (Chebyshev-Halley method) and the method of tangent parabolas (Euler-Chebyshev method). A family of iterative methods having the convergence order equal to four is also considered. Convergence properties and computational aspects of the methods under consideration are examined. The problems of their global implementation and polyalgorithmic strategy are discussed as well.

Keywords: Banach space, large scale optimization, hierarchical decision making, decomposition-coordination schemes, the method of tangent hyperbolas, the method of tangent parabolas, global convergence.

1. Introduction

A number of problems in economics, engineering and scientific computation (e.g. production planning, process control, image restoration, parameter identification, neural networks, inverse problems) lead frequently to a large mathematical programming problem

$$\min\{f(x) : x \in Q\}, \tag{1}$$

where Q is a closed subset of $\Re^n$. It also contains the problem of finding fixed points of nonlinear mapping F, i.e.

$$F(x) = 0 \tag{2}$$

with

$$f(x) = \frac{1}{2}\|F(x)\|^2 \quad \text{and} \quad Q = \Re^n, \tag{3}$$

where F is acting between spaces $\Re^n$ and $\in \Re^m$. On the other hand, the problem of finding an extremum for constrained optimization problems is frequently reduced by means of Lagrange multipliers or penalty functions to seeking stationary points of certain unconstrained functionals. Thus problems (1) and (2) are closely related.

One of the potential ways to reduce the total time needed for computing the solution of a large scale problem is to use parallel computations. The idea of hierarhical decision making is to reduce the overall complex problem into smaller and simpler approximate problems which can then be distributed over a larger number of processors and treated independently. One way to break a large problem into smaller subproblems is the use of decomposition-coordination schemes, i.e. by designating the processors (computers) as the master and slaves. Decomposability is a kind of organized sparseness. The problem variables can be divided into groups so that most variables interact only with members of their own group. It should then be possible to solve the problem hierarchically: on the top level, setting values for a small number of variables common to the groups, and on the lower level, solving independently within each group for those variables that interact only with others in the same group.

The computation of proper values for coordination parameters in convex programming leads often to solving an auxiliary optimization problem or a system of nonlinear equations

$$H(y(\beta),\beta) = 0, \tag{4}$$

where $H = (H_1,...,H_m)^T$ and $\beta = (\beta_1,...,\beta_m)^T$ while the components of vector $y = (y_1,...,y_m)^T$ are to be determined as solution of nonlinear problems

$$F_i(y_i,\beta) \to \min, \quad y_i \in \Gamma_i(\beta) \tag{5}$$

depending on parameter vector β and where F_i is the performance index of i-th subproblem and $\Gamma_i(\beta) \subset \Re^{n_i}$ is its feasible region.

In order to develop many optimum plans and to treat with large scale problems a multi-level approach may be useful.

Arguably, many socio-ecological and industrial optimization problems are of a multiobjective type. If different decision makers stand behind different objective functions $F_1(x),...,F_k(x)$, then the two-level strategy based on the weighting method can be used. In the weighting method the idea is to associate each objective function with a weighting factor and to minimize the weighted sum of objectives. In this way the multiple objective functions are transformed into a single objective function:

$$\text{minimize } \sum_{i=1}^{k} w_i f_i(x),$$

$$\text{subject to } x \in Q,$$

$$\text{where } w_i \geq 0 \text{ for all } i = 1,...,k \text{ and}$$

$$\sum_{i=1}^{k} w_i = 1.$$

Therefore, the weighting method can be considered as a two-level optimization procedure: on the first level we find proper weights w_i and after that we minimize $\sum_{i=1}^{k} w_i f_i(x)$.

Assuming that the problems (5) have the solutions $y_i = y_i(\beta^*)$, we shall study the problem for determining coordination vector $\beta^* \in \Re^m$ from the equation (4).

Further on, we shall reformulate the problem (4) into the form (2) more habitual for mathematics, where $F(x) \equiv H(\cdot, x)$ and x will stand for the argument.

Decomposition-coordination problems have some specific features:

- the user has at his/her disposal only functional values;
- the evaluation of functional values includes, basically, the solution of certain subproblems and therefore it can cause a great computational effort;
- the functions involved are not necessarily differentiable, they may belong to a set of almost differentiable functions.

Besides the problem (4) may be ill-conditioned or even ill-posed, i.e. we cannot assume the existence of $(F')^{-1}$ or its boundedeness.

Therefore, when using multi-level approach for problem solving sophisticated algorithms are needed which try to find trade-off between robustness, stability and efficiency. Methods with the high order of convergence making full use of local information (e.g. functional values, gradient and Hessian) permit sometimes to win in speed and accuvacy.

Computational effort is often one of the basic problems in the solution of real-world problems. The total cost of an iterative method is determined by the number of iterations needed to achieve the required accuracy and the cost of each iteration. The implementation of methods with the high order of convergence requires computing the solution with the prescribed accuracy, as a rule, less iterations than the methods with a lower convergence order and therefore likely less total arithmetic.

As for stability which is another important aspect of computation the use of methods with the high order of convergence may relieve the stability problem as well because they are based at least on a quadratic model. As shown in [1, 2] even very rough approximation to the operator of the second derivatives in the methods with the convergence order $p = 3$ may provide their numerical stability.

But the functions involving in decomposition-coordination schemes may be nonsmooth therefore in recent years nonsmooth Newton and smoothing methods for solving semismooth and piecewise smooth equations have received much attention [3, 4]. One possibility to handle equations with nonsmooth functions is to approximate the locally Lipschitzian function with a smooth one and to use the derivative of the smooth function in the algorithm whenever a derivative is needed (e.g. in an extension of the Levenberg-Marquardt method as suggested in [4]). Certainly, in the regions where operator F is nonsmooth the use of methods with a lower convergence order (e.g. secant type methods [5]) may be more effective.

2. Methods

For finding solutions of (4) we consider approximate variants of high order methods of the type

$$x_{k+1} = x_k - Q(x_k, A_k), \quad k = 0, 1, ..., \tag{6}$$

where $Q(x, A_k)$ is a sufficiently many times Frechet-differentiable operator from Banach X space into itself. It is assumed that there exists an exact method which is known to be convergent with the convergence order $p \geq 2$ and has a similar form. The study of methods with approximate operators gives a more relevant impression of the methods under discussion. Frequently the use of finite-difference approximations to the derivatives gives rise to an inexact method. An approximate variant of the method can also be obtained as a result of strategy used for solving linear problems at each iteration, i.e. the associated linear equations are solved approximately by taking finitely many steps of an iterative procedure or the inverse operators are approximated by a recurrence formula.

If $Q(x_k, A_k) = A_k F(x_k)$ and $A_k \approx \Gamma_k = \left[F'(x_k)\right]^{-1}$

one gets a Newton-like method. Let I denote the identity mapping.

If

$$Q(x_k, A_k) = \left[I - \frac{1}{2} A_k F''(x_k) A_k F(x_k)\right]^{-1} A_k F(x_k), \quad (7)$$

$$Q(x_k), A_k) = A_k F(x_k) + \frac{1}{2} A_k F''(x_k)(A_k F(x_k))^2. \quad (8)$$

Approximate variants of the cubically convergent method of tangent hyperbolas (Chebyshev-Halley method) and the cubically convergent method of tangent parabolas (Euler-Chebyshev) method are obtained respectively. If in (7) instead of $-F''(x_k) A_k F(x_k)$ we use the approximations

$$2\beta\left[F'(x_k - \frac{1}{2\beta} A_k F(x_k)) - F'(x_k)\right], \beta \neq 0,$$

then we obtain

$$x_{k+1} =$$

$$= x_k - \left[(1-\beta)F'(x_k) + \beta F'(x_k - \frac{1}{2\beta} A_k F(x_k))\right]^{-1} F(x_k) \quad (9)$$

which coincides with the midpoint method

$$x_{k+1} =$$

$$= x_k - \left[F'(x_k - \frac{1}{2}\Gamma_k F(x_k))\right]^{-1} F(x_k) \quad (10)$$

provided $\beta = 1$ and $A_k = \Gamma_k$.

To get a derivative free method the formula (10) can be modified as follows:

$$x_{k+1} =$$

$$= x_k - [F(2u_k - x_k; x_k]^{-1} F(x_k), \quad (11)$$

where $F(v; w)$ denotes the first order divided difference with basic elements v and w, $u_k = x_k - \frac{1}{2} \mathrm{B}_k F(x_k)$ and

$$\mathrm{B}_k = [F(2x_k - x_{k-1}; x_k)]^{-1}.$$

Another possibility to avoid the evalution of F'' and thereby to reduce computational efforts is to replace it by a fixed bilinear operator:

$$x_{k+1} =$$

$$= x_k - \left[I - \frac{1}{2} A_k \Phi A_k F(x_k)\right] A_k F(x_k), \quad (12)$$

where Φ is a general bounded bilinear operator. The execution of one iteration step by the formula (12) is equivalent to solving two perturbed linear equations

$$\left[F'(x_k) + V_k\right](y_k - x_k) = -F(x_k),$$

$$\left[F'(x_k) + V_k\right](x_{k+1} - y_k) =$$

$$= -\frac{1}{2}\Phi(y_k - x_k)^2,$$

where $V_k = A_k^{-1} - F'(x_k)$. Therefore (12) has the similar computational costs as Newton method. It can be shown that (12) with $A_k = \Gamma_k$ remains faster than Newton method [6].

Using in (8) the approximation

$$F''(x_k)(A_k F(x_k))^2 \approx$$

$$\approx 2\left[F(x_k - A_k F(x_k)) - F(x_k) - A_k^{-1}(x - A_k F(x_k)) - x_k\right]$$

we get an approximate variant of Euler-Chebyshev method

$$x_{k+1} =$$

$$= x_k - A_k F(x_k) - A_k F(x_k - A_k F(x_k)). \quad (13)$$

This method is remarkable in the sense that in order to guarantee the convergence order $p = 3$ for (8) one has to solve the corresponding linear problems with the accuracy $O(\|F(x_k)\|^2)$, while for its variant (13) the accuracy of approximation $O(\|F(x_k)\|)$ is sufficient to obtain the same convergence order.

When only functional values are available then inexact methods based on finite-difference approximations are greatly useful. The derivative-free variant of (13)

$$y_k = x_k - [F(2y_{k-1} - x_{k-1}; x_{k-1})]^{-1} F(x_k),$$

$$x_{k+1} = y_k - [F(2y_k - x_k; x_k)]^{-1} F(y_k) \quad (14)$$

has the asymptotic convergence order equal to 3 provided F'' and its divided difference are Lipschitz continous.

If it is not difficult to evaluate the derivatives of F then iterative methods with more high convergence order may be used, e.g.

$$x_{k+1} = v_k - 2AF(v_k) -$$

$$-\frac{1}{\rho}\bar{A}_k[F(v_k + \rho A_k F(v_k)) - F(v_k)],$$

$$v_k = x_k - \tilde{A}_k F(x_k), \tag{15}$$

where $A_k, \bar{A}_k, \tilde{A}_k \approx \Gamma_k$ with $\|I - F'(x_k)A_k\| \le \chi_k < 1$, $\|I - F'(x_k)\bar{A}_k\| \le \bar{\chi}_k < 1$, $\|I - F'(x_k)\tilde{A}_k\| \le \tilde{\chi}_k < 1$, and ρ is a nonzero parameter.

Under the assumption that F'' is Lipschitz-continuos it can be shown that the iterative process (15) has the convergence order equal to 4 provided $\chi_k, \bar{\chi}_k, \tilde{\chi}_k$ are of order $O(\|F(x_k)\|)$ and $\|A_k - \bar{A}_k\| = O(\|F(x_k)\|^2)$ [7].

If $A_k = \bar{A}_k = \tilde{A}_k$ and $\chi_k = O(\|F(x_k)\|)$ then from (15) we get the fourth order method

$$x_{k+1} = v_k + (A_k F'(v_k) - 2I)A_k F(v_k)$$

$$v_k = x_k - A_k F(x_k) \tag{16}$$

provided $\rho \to 0$.

3. Convergence theorem

Assume that the uniformly bounded inverse operator exists as well as the constants M, K, λ, d and the sequence $\{\chi_k\}$ satisfying the following inequalities is:

$$\|F'(x)\| \le M,\ \|F''(x)\| \le K,\ \|A_k\| \le \Lambda,$$

$$\|x_{k+1} - x_k\| \le \lambda\|F(x_k)\|,\ (\lambda, \Lambda < \infty) \tag{17}$$

$$\|I - F'(x_k)A_k\| \le \chi_k,\ \|F(x_{k+1})\| \le d\|F(x_k)\|^3,$$

$$k = 0,1,... \tag{18}$$

Theorem. *Let* $x_0 \in X$, $S = \{x \in X : \|x - x_0\| \le \rho\}$ *and let the following conditions be valid on S:*

1º operator F is twice Frechet-differentiable;

2º the second derivative satisfies a Lipschitz-condition

$$\|F''(x) - F''(y)\| \le L_2\|x - y\|;\ 0 < L_2 < \infty;$$

3º there exists $\Gamma(x)$ *with* $\|\Gamma(x)\| \le C$ *and* $C < \infty$;

4º $\delta = \sqrt{d}\|F(x_0)\| < 1$.

Then the following results are valid:

If $\chi_k \le C_2\|F(x_k)\|$, $C_2 < \infty$, $r_3 = \lambda H_0^{(3)}(\delta)/\sqrt{d} \le \rho$,

where $H_k^{(3)}(\delta) = \sum_{i=k}^{\infty} \delta^{3^i}$, $\delta = \delta_0 = \sqrt{d}\|F(x_0)\| < 1$.

and $d = C_2 + w_1 C_2 + w_2$,

then the sequence (13) converges cubically

$$\|x_k - x^*\| \le (\lambda/\sqrt{d})H_k^{(3)}(\delta).$$

Proof. Letting w_1 and w_2 be positive constants with $w_1, w_2 < \infty$ we shall first show the validity of the following inequality

$$\|F(x_{k+1})\| \le \chi_k^2\|F(x_k)\| +$$

$$+ w_1\chi_k\|F(x_k)\|^2 + w_2\|F(x_k)\|^3. \tag{19}$$

Indeed, taking $Q_1(x) = x$ then

$$Q_2(x) = Q_1(x) - \Gamma(x)F(x)$$

generates Newton method and, in general, $p \ge 2$

$$Q_{p+1}(x) = Q_p(x) - \Gamma(x)F\left[Q_p(x)\right] \tag{20}$$

defines an iterative method

$$x_{k+1}^{(p+1)} := Q_{p+1}(x_k) = Q_p(x_k) - \Gamma(x_k)F\left[Q_p(x_k)\right] \tag{21}$$

having the convergence order equal to $p+1$.

Replacing Γ_k in (21) by its approximation on the basis of Taylor expansion

$$F(x + \Delta x) = F(x) + F'(x)\Delta x +$$

$$+\int_0^1 F''(x + t\Delta x)\Delta x^2(1-t)dt$$

we have

$$\|F(x_{k+1}^{(2)})\| \le \|\left[I - F'(x_k)A_k\right]F(x_k) +$$

$$+\int_0^1 F''(x_k - tA_kF(x_k))\left[A_kF(x_k)\right]^2(1-t)dt\| \le$$

$$\le \chi_k\|F(x_k)\| + \frac{1}{2}\Lambda^2 K\|F(x_k)\|^2. \tag{22}$$

Note, that in the capacity of Λ and K we can take $\Lambda = C(1+\chi_0)$ and $K = \|F''(x_0)\| + L_2\rho$ respectively. In analogy we have

$$\|F(x_{k+1}^{(3)})\| = \|F(x_{k+1}^{(2)} - F'(x_{k+1}^{(2)})A_kF(x_{k+1}^{(2)}) +$$

$$+\int_0^1 F''(x_{k+1}^{(2)}) - \tau A_kF(x_{k+1}^{(2)})\left[A_kF(x_k)\right]^2(1-\tau)d\tau\| \le$$

$$\leq \left\| I - F'(x_{k+1}^{(2)})A_k \right\| \left\| F(x_{k+1}^{(2)}) \right\| + G \left\| F(x_{k+1}^{(2)}) \right\|^2$$

with $G < \infty$. Taking $x_{k+1} := x_{k+1}^{(3)}$ and bearing in mind (21) we get

$$\| x_{k+1} - x_k \| = \left\| x_{k+1}^{(3)} - x_k \right\| \leq$$

$$\leq \| A_k F(x_k) \| + \left\| A_k F(x_{k+1}^{(2)}) \right\| \leq$$

$$\leq \lambda_k \| F(x_k) \| \leq \lambda \| F(x_k) \|,$$

where $\lambda = \left[1 + \chi_0 + \frac{1}{2} \Lambda^2 K \| F(x_0) \| \right]$. On the basis of (21) and (22) it is not hard to obtain the inequality (19). Since $\chi_k \leq C_2 \| F(x_k) \|$ then

$$\| F(x_{k+1}) \| \leq d \| F(x_k) \|^3,$$

where $d = C_2 + w_1 C_2 + w_2)$.

Thus

$$\| x_n - x_k \| \leq \sum_{m=k}^{n-1} \| x_{m+1} - x_m \| \leq$$

$$\leq \lambda d^{-\frac{1}{2}} \left[H_k^{(3)}(\delta) - H_n^{(3)}(\delta) \right]$$

with $n \geq k$, i.e. the sequence $\{x_k\}$ is fundamental and consequently

$$x^* = \lim_{k \to \infty} x_k,$$

$$\left\| x^* - x_k \right\| \leq \lambda d^{-\frac{1}{2}} H_k^{(3)}(\delta) \leq \rho,$$

$$\left\| x_0 - x^* \right\| \leq \lambda d^{-\frac{1}{2}} H_0^{(3)}(\delta).$$

Remark. It is shown in [6] that for Euler-Chebyshev method

$$\| F(x_{k+1}) \| \leq \chi_k \| F(x_k) \| +$$

$$+ w_1 \chi_k \| F(x_k) \|^2 + w_2 \| F(x_k) \|^3$$

and therefore the rate of approximation

$\chi_k \leq C_2 \| F(x_k) \|^2$ may only quarantee the cubic convergence.

4. Computational aspects

For today there are lots of methods with $p \geq 2$, but in practice they are relatively little exploited. This is partially due to the fact that computational schemes of execution of one iteration of these methods are laborous, they require frequently the evaluation of derivatives of order greater than one and a good initial guess since their advantages become evident in the close vicinity of the solution. On the other hand the total cost of an iterative method is determined by the number of iterations needed to achieve the required accuracy and the cost of each iteration. The implementation of methods with the high order convergence requires the solution with the prescribed accuracy for computing as a rule less iterations than the methods with a lower convergence order and therefore likely less iterations.

The property of global convergence is a criterion for robustness. One of the popular ways to guarantee the global convergence or at least to expand the domain of convergence is the "continuation strategy". According to this idea, firstly the equation $F(x) = 0$ must be replaced by a one-parameter family of problems $G(x\lambda) = 0$, $\lambda \in [0,1]$, such that $F(x) = G(x,1)$ and the solution of $G(x,0) = 0$ is known. Secondly, a series of problems must be solved, where parameter λ is slowly varied. But all the homotopy methods suffer from the disadvantage that the Jacobian may at some iteration points become singular. One more reason for using methods with the convergence order greater than that of Newton method is the fact that methods with the convergence order $p > 2$ do not break down, if F' is singular or strongly ill-conditioned since they are based, at least, on a quadratic model and even very rough approximation to the operator of second derivatives may provide their numerical stability [1, 2]. Recall that continuous methods converge globally but slowly, whereas the iterative methods with a large order of convergence converge locally. These features of methods can be combined in such a way that the continuation method is used, if necessary, to help get into the domain of convergence of the rapidly convergent method which then will be turned on to improve the accuracy.

As mentioned before the functions involved in decomposition-coordination schemes may be nonsmooth. One possibility to handle equations with nonsmooth functions is to approximate the locall Lipschitzian function with a smooth one and to use the derivative of the smooth function in the algorithm whenever a derivative is needed.

The trust region method is one of the effective ways to compose polyalgorithmic computational schemes. In the paper [4] a trust region method for solving nonsmooth equations subject to linear constraints is proposed.

The performance of methods if the type (6) is equivalent to either solving the associated linear equations or computing the inverses with an error at every iteration step. A strategy of problem solving that instead of finding the exact solution of a linear problem at every iteration solves it intentionally inexactly is a possibility to save computational work and is adaptive in the sense that one uses low accuracy numerical solution at inner iterations when the solution is not reached yet and improves the accuracy as the solution is approached. In many cases iterative methods are more appropriate and economical for solving linear problems than direct ones. Thus, the strategy to solve the corresponding linear problems intentionally inexactly can be used for the purpose of economy. Besides, iterative methods are self-corecting and hence they are not sensitive to computational errors.

Multi-criteria analysis uses evaluations on several criteria to recommend a decision. Any decision might be "better" than other depending on the point of view. Within the context of the "compromise" principle classical economics asserts that market economy, through competetive equilibrium, ensures the most efficient allocation of resources judged by Pareto optimality criterion that no one can be made better off without making some one worse off. In reality markets may fail to allocate resources in an optimum way (market failures) and government intervention under these conditions is justified as a means for achieving efficient allocation of resources among societal objectives [7].

5. Conclusion

Although we have disscussed here the methods on the theoretical basis, numerical experience with the methods under consideration has also confirmed these theoretical considerations. The performance of the combination of the cubically convergent method (14) with Newton method was superior both in speed and accuracy than single Newton method. These result can partially be found in [6]. These promising results encourage us to carry out the investigation of properties of polyalgorithmic procedures.

6. Acknowledgement

The support of the Estonian Science Foundation under grant Nr. 5006 is gratefully achnowledged.

References

1. Ehle, G. P.; Schwetlick, H. Discretized Euler-Chebyshev multistep method. *SIAM J. Numer. Anal.*, Vol 13, No 3, 1976, p. 433–443.
2. Döring, B. Iterative Lösung Gewisser Randwertprobleme und Integralgleichungen. *Applicae Matematiky*, Vol 23, 1978, p. 1–31.
3. Li Qun Qi. Trust region algorithms for solving nonsmooth equations. *SIAM J. Optim.,* Vol 5, No 1, 1995, p. 219–230.
4. Gabriel, A.; Jong-Shi, P. A trust region method for constrained nonsmooth equations. In: Large Scale Optimization: State of the Art (eds. W.W. Hagerer et al). Kluwer Academic Publishers. Dordrecht, 1994, p. 155–181.
5. Amat, S.; Busquier, S.; Gutierrez, J. M. On the local convergence of secant-type methods. *Intern. J. Comp. Math.*, Vol 81, No 9, 2004, p. 1153–1161.
6. Vaarmann, O. On high order iterative methods for nonlinear problems. In: Nonlinear Mathematical Analysis and Applications (ed. Th. M. Rassias). Palm Harbor, FL; Hadronic Press, 1998, p. 275–299.
7. Sardar, M. N. Islam Mathematical Economics of Multi-Level Optimisation. Physica Verlag, A Springer-Verlag Company, 1997. 256 p.

DAUGIALAIPSNIAI ITERACINIAI METODAI SKAIDYMO IR JUNGIMO PROBLEMOMS SPRĘSTI

O. Vaarmann

Santrauka

Daugelis realių optimizavimo uždavinių yra daugiatiksliai ir daugiadimensiai. Straipsnyje nagrinėjamos sudėtingų optimizacijos uždavinių sprendimų galimybės daugialaipsniu metodu, naudojant kompiuterinį tinklą arba daugiaprocesorinį kompiuterį. Apžvelgiami tokie įprastininiai skaitiniai metodai, kaip matematinis modeliavimas, kuriame neišvegiama paklaidų apvalinimo. Skaidymo ir jungimo problemoms spręsti, remiantis tangentinės hiperbolės (Čebyševo ir Halėjaus) ir tangentinės parabolės (Oilerio ir Čebyševo) metodais, sukurti keli greitos konvergencijos interaciniai metodai. Taip pat aptariama ketvirtojo laipsnio konvergencijos metodų šeima. Nagrinėjamos sukurtųjų metodų konvergavimo savybės ir skaičiavimo jais aspektai. Svarstomos pasiūlytųjų metodų ir polialgoritminės strategijos visuotinio taikymo galimybės.

Pagrindiniai žodžiai: Banacho erdvė, daugiatikslė optimizacija, hierachinis sprendimų priėmimas, skaidymo ir jungimo schemos, tangentinės hiperbolės ir tangentinės parabolės metodai, globalinė konvergencija.

Otu VAARMANN Associate Professor, Tallinn University of Technology. Doctor of Science (1970) of the Estonian Academy of Sciences. Additional studies: Research Associate at the Glushkov Instiute of Cybernetics (Kiew) (1964-1965); Ph.D studies at the Institute of Cybernetics (1965-1968). Research interests: Methods for nonlinear equations and optimization problems, pseudoinversion, ill-posed problems, hiearchical decision making.