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SIMPLE METHODS OF ENGINEERING CALCULATION FOR SOLVING HEAT TRANSFER PROBLEMS

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ABSTRACT

There are well-known numerical methods for solving the initial-boundary value problems for partial differential equations. We mention only some of them: finite difference method (FDM), finite element method (FEM), boundary element method (BEM), Galerkin type methods and others. In the given work FDM and BEM are considered for determination a distribution of heat in the multilayer media. These methods were used for the reduction of the 1D heat transfer problem described by a partial differential equation to an initial–value problem for a system of ordinary differential equations (ODEs). Such a procedure allows us to obtain a simple engineering algorithm for solving heat transfer equation in multilayered domain. In a stationary case the exact finite difference scheme is obtained. An inverse problem is also solved. The heat transfer coefficients are found and temperatures in the interior layers depending on the given temperatures inside and outside of a domain are obtained.

Key words: transfer problems, initial-boundary value problems, partial differential equations, finite difference method, finite element method

1. THE MATHEMATICAL MODEL

We shall consider the partial differential equation

$$c\rho\frac{\partial u}{\partial t} = \frac{\partial}{\partial z}\Big(\lambda\frac{\partial u}{\partial z}\Big) + q(z,t), \tag{1.1}$$

where c is a specific thermal capacity $(\frac{J}{kg.K})$, ρ is density $(\frac{kg}{m^3})$, λ is the coefficient of heat conductivity $(\frac{W}{m.K})$, q is the function of thermal sources $(\frac{W}{m^3})$, t is the time (s) and u is the absolute temperature (K).

Multilayer media Ω consists of N layers

$$\Omega = \{z : z \in \Omega_k, \quad k = \overline{1,N}\},$$

where each layer is characterized by set $\Omega_k = \{z : z_{k-1} \le z \le z_k\}$, and $z = z_k, k = \overline{1, N-1}$ are the joint of the layers (the interior grid points in the FDM).

If every layer has parameters $\rho_k, c_k, \lambda_k, q_k$, then the heat equation (1.1) can be presented in the following form

$$\frac{\partial}{\partial z}\Big(\lambda_k \frac{\partial u_k}{\partial z}\Big) = F_k, \qquad k = \overline{1,N}, \tag{1.2}$$

where

$$F_k = c_k \rho_k \frac{\partial u_k}{\partial t} - q_k(z,t)$$

and $u_k = u_k(z,t)$ is the temperature in the layer Ω_k.

We have the following conditions:

1) Continuity conditions on surfaces $z = z_k, k = \overline{1, N-1}$:

$$u_k(z_k,t) = u_{k+1}(z_k,t),$$

$$\lambda_k \frac{\partial u_k(z_k,t)}{\partial z} = \lambda_{k+1} \frac{\partial u_{k+1}(z_k,t)}{\partial z};$$

2) boundary conditions on the surfaces $z = z_0$ and $z = z_N$:

$$\begin{cases} \lambda_1 \dfrac{\partial u_1(z_0,t)}{\partial z} - \alpha_1(u_1(z_0,t) - \theta_0) = 0, \\ \\ \lambda_N \dfrac{\partial u_N(z_N,t)}{\partial z} + \alpha_N(u_N(z_N,t) - \theta_N) = 0, \end{cases} \tag{1.3}$$

where α_1, α_N are convection heat transfer coefficients, θ_0, θ_N are the dimensionless temperatures of air. For the initial conditions we give the temperature $u_k(z,0)$ in every layer $k = \overline{1,N}$.

2. THE FINITE VOLUMES METHOD AND THE FDM

Using the finite volumes method [1; 2; 3], we obtain the finite-difference scheme, i.e. $N+1$ equations are given on a joint of layers

$$\begin{cases} \frac{\lambda_1}{h_1}(u_1 - u_0) - \alpha_1(u_0 - \theta_0) = R_0^+, \\ \frac{\lambda_{k+1}}{h_{k+1}}(u_{k+1} - u_k) - \frac{\lambda_k}{h_k}(u_k - u_{k-1}) = R_k^- + R_k^+, \ 1 \le k < N, \\ -\alpha_N(u_N - \theta_N) - \frac{\lambda_N}{h_N}(u_N - u_{N-1}) = R_N^-, \end{cases} \tag{2.1}$$

where

$$\begin{aligned} R_k^- &= I_k^- + \tilde{R}_k^-, \quad R_k^+ = I_k^+ + \tilde{R}_k^+, \\ \tilde{R}_k^- &= \frac{c_k \rho_k}{h_k} \int_{z_{k-1}}^{z_k} (z - z_{k-1}) \dot{u}_k(z,t) dz, \ k = \overline{1, N}, \\ \tilde{R}_k^+ &= \frac{c_{k+1}\rho_{k+1}}{h_{k+1}} \int_{z_k}^{z_{k+1}} (z_{k+1} - z) \dot{u}_{k+1}(z,t) dz, \ k = \overline{0, N-1}, \\ \dot{u}_k(z,t) &= \frac{\partial u_k(z,t)}{\partial t}, \quad h_k = z_k - z_{k-1}, \quad k = \overline{1, N}, \\ I_k^- &= -\frac{1}{h_k} \int_{z_{k-1}}^{z_k} (z - z_{k-1}) q_k(z,t) dz, \ k = \overline{1, N}, \\ I_k^+ &= -\frac{1}{h_{k+1}} \int_{z_k}^{z_{k+1}} (z_{k+1} - z) q_{k+1}(z,t) dz, \ k = \overline{0, N-1}. \end{aligned}$$

In a stationary case exactly calculating integrals I_k^-, I_k^+ we obtain the exact finite-difference scheme.

3. THE BEM AND THE FINITE-DIFFERENCE SCHEME

The finite difference scheme (2.1) can be obtained by using the BEM. Applying this method for equation (1.2) in the segment $[z_{k-1}, z_k]$, multiplying equation (1.2) by the function $w(z,\xi) = |z - \xi|, \xi \in [z_{k-1}, z_k]$ and integrating it by parts twice we get

$$\lambda_k \int_{z_{k-1}}^{z_k} u_k w'' dz = \int_{z_{k-1}}^{z_k} F_k w dz + \lambda_k P_k, \tag{3.1}$$

where

$$P_k = (u_k w' - u_k' w)\Big|_{z_{k-1}}^{z_k}, \quad w' = \frac{\partial w}{\partial z}.$$

Due to equalities $w' = sign(z-\xi)$, $w'' = \delta(z-\xi)$ (here $\delta(z-\xi)$ is the Dirac-delta function) we obtain the third Green formula for the 1D case:

$$\lambda_k u_k(\xi,t) = \int_{z_{k-1}}^{z_k} |z-\xi| F_k dz + \lambda_k P_k, \tag{3.2}$$

where

$$\begin{aligned} P_k &= v_k(z_k) sign(z_k-\xi) - v_k'(z_k)|z_k-\xi| - v_k(z_{k-1}) sign(z_{k-1}-\xi) \\ &\quad + v_k'(z_{k-1})|z_{k-1}-\xi|, \quad v_k(z) \equiv u_k(z,t). \end{aligned}$$

From (3.2) for the given values $v_k(z_k), v_k(z_{k-1}), v_k'(z_k), v_k'(z_{k-1}), F_k$ it is possible to find $v_k(\xi) \equiv u_k(\xi,t)$ for all $\xi \in [z_{k-1}, z_k]$.

Let us consider two limit cases, when $\xi \to z_{k-1}$ and $\xi \to z_k$. Then we have two equations in the following form:

$$\begin{cases} \lambda_k v_k(z_{k-1}) = \lambda_k (v_k(z_k) - h_k v_k'(z_k)) + h_k R_k^-, \\ \\ \lambda_k v_k(z_k) = \lambda_k (v_k(z_{k-1}) + h_k v_k'(z_{k-1})) + h_k R_{k-1}^+. \end{cases} \tag{3.3}$$

Substituting k by $k+1$ in the second equation (3.3), then dividing these expressions respectively by h_k, h_{k+1} and applying the continuity conditions

$$v_k(z_k) = v_{k+1}(z_k), \quad \lambda_k v_k'(z_k) = \lambda_{k+1} v_{k+1}'(z_k), \quad k = \overline{1, N-1}$$

we obtain the difference equations (2.1) for $k = \overline{1, N-1}$. The first equation of (2.1) is obtained from the second equation (3.3), if $k = 1$, and the last equation – from the first equation (3.3), if $k = N$ (the boundary conditions (1.3) must be used).

From the finite difference scheme (2.1) we obtain the values $u_{k-1}, u_k, k = \overline{1, N}$, and from (3.3) – the values $v_k'(z_{k-1}), v_k'(z_k)$ in the interior grid points.

4. THE ANALYTICAL SOLUTION IN A STATIONARY CASE

In this case the difference scheme can be solved analytically. Representing the first equation of (2.1) in the form

$$A_1(u_1 - u_0) - \alpha_1^+(u_1 - \theta_0) = R_0^+ \frac{\alpha_1^+}{\alpha_1}$$

and adding to the second one, we obtain

$$A_1(u_2 - u_1) - \alpha_1^+(u_1 - \theta_0) = \alpha_1^+ Q_1^+, \tag{4.1}$$

where

$$A_k = \frac{\lambda_k}{h_k}, \quad k = \overline{1, N},$$
$$Q_1^+ = \frac{R_1}{\alpha_1^+} + Q_0^+, \quad Q_0^+ = \frac{R_0}{\alpha_1},$$
$$R_k = R_k^+ + R_k^-, \quad k = \overline{1, N-1}, \quad R_0 = R_0^+,$$

α_1^+ is the factor of thermal resistance of two layers numbering the layers in the growing sequence. We find, similarly to the formula for total resistance in parallel electric circuits, that

$$\frac{1}{\alpha_1^+} = \frac{1}{\alpha_1} + \frac{1}{A_1}.$$

After this step we find the equation

$$A_{m+1}(u_{m+1} - u_m) - \alpha_m^+(u_m - \theta_0) = \alpha_m^+ Q_m^+, \tag{4.2}$$

where α_m^+ is the common factor of thermal resistance of $(m+1)$ layers,

$$\frac{1}{\alpha_m^+} = \frac{1}{\alpha_{m-1}^+} + \frac{1}{A_m} = \sum_{i=0}^{m} \frac{1}{A_i}, \qquad A_0 = \alpha_1,$$
$$Q_m^+ = \frac{R_m}{\alpha_m^+} + Q_{m-1}^+ = \sum_{i=0}^{m} \frac{R_i}{\alpha_i^+} \qquad (\alpha_0^+ = \alpha_1,\ R_0 = R_0^+).$$

Similarly expressing unknowns $u_N, u_{N-1}, \ldots$ in an opposite direction, after n steps we find

$$\alpha_{N-n}^-(\theta_N - u_{N-n}) - A_{N-n}(u_{N-n} - u_{N-n-1}) = \alpha_{N-n}^- Q_{N-n}^-, \tag{4.3}$$

where α_{N-n}^- is the common factor of the thermal resistance (here n is numbered in a backward direction)

$$\frac{1}{\alpha_{N-n}^-} = \frac{1}{\alpha_{N-n+1}^-} + \frac{1}{A_{N-1}} = \sum_{i=0}^{n} \frac{1}{A_{N-i+1}}, \qquad A_{N+1} = \alpha_N,$$
$$Q_{N-n}^- = \frac{R_{N-n}}{\alpha_{N-n}^-} + Q_{N-n+1}^- = \sum_{i=0}^{n} \frac{R_{N-i}}{\alpha_{N-i}^-}$$
$$\alpha_N^- = \alpha_N, \quad R_N = R_N^-, \quad Q_N^- = R_N/\alpha_N).$$

Taking $n = N - k, m = k - 1$ in (4.2), (4.3) we obtain the system of two equations

$$\begin{cases} A_k(u_k - u_{k-1}) - \alpha_{k-1}^+(u_{k-1} - \theta_0) = \alpha_{k-1}^+ Q_{k-1}^+ \\ \alpha_k^-(\theta_N - u_k) - A_k(u_k - u_{k-1}) = \alpha_k^- Q_k^-, \qquad k = \overline{1,N}. \end{cases} \tag{4.4}$$

Representing the first equation in the form

$$A_k(u_k - u_{k-1}) - \alpha_k^+(u_k - \theta_0) = \alpha_k^+ Q_{k-1}^+$$

and adding to the second equation, we find

$$u_k = \frac{\alpha_k^- \theta_N + \alpha_k^+ \theta_0 - Q_k}{\alpha_k^+ + \alpha_k^-}, \tag{4.5}$$

where $Q_k = Q_k^- \alpha_k^- + Q_{k-1}^+ \alpha_k^+, \; k = \overline{1,N}$.

If $n = N - 1$, then it follows from (4.3), that

$$\alpha_1^-(\theta_N - u_1) - A_1(u_1 - u_0) = \alpha_1^- Q_1^-$$

or

$$\alpha_0^-(\theta_N - u_0) - A_1(u_1 - u_0) = \alpha_0^- Q_1^-.$$

Therefore

$$u_0 = \frac{\alpha_0^- \theta_N + \alpha_1 \theta_0 - Q_0}{\alpha_1 + \alpha_0^-}, \tag{4.6}$$

where $Q_0 = Q_0^- \alpha_0^- + R_0$.

If $q_k \equiv 0$, then the stationary solution is the linear function in each layer, and it can be represented in the following form:

$$u_k^{stac}(z) = \frac{z - z_k}{z_{k-1} - z_k} u_{k-1} + \frac{z - z_{k-1}}{z_k - z_{k-1}} u_k$$
$$= \frac{z - z_{k-1}}{h_k} u_k - \frac{z - z_k}{h_k} u_{k-1}), \; z \in [z_{k-1}, z_k], \; k = \overline{1,N}.$$

If $\alpha_1 = 0$, then in formulae (4.5), (4.6) it is necessary to substitute $\alpha_k^+ = 0$, and $Q_{k-1}^+ \alpha_k^+ = R_0 + R_1 + ... + R_{k-1}$.

In the case of the Dirichlet boundary condition the first and the last equations (2.1) don't depend on $\alpha_1; \alpha_N \to \infty$ and $u_0 = \theta_0, \;\; u_N = \theta_N$.

In the case of the Neuman boundary condition it is necessary to substitute $\alpha_1 = \alpha_N = 0$.

As an example, it follows from (4.5), (4.6) for $N = 3$

$$\begin{cases} u_0 = \dfrac{\alpha_0^-\theta_3 + \alpha_1\theta_0 - Q_0}{\alpha_1 + \alpha_0^-}, & u_1 = \dfrac{\alpha_1^-\theta_3 + \alpha_1^+\theta_0 - Q_1}{\alpha_1^+ + \alpha_1^-}, \\ u_2 = \dfrac{\alpha_2^-\theta_3 + \alpha_2^+\theta_0 - Q_2}{\alpha_2^+ + \alpha_2^-}, & u_3 = \dfrac{\alpha_3\theta_3 + \alpha_3^+\theta_0 - Q_3}{\alpha_3^+ + \alpha_3}, \end{cases} \tag{4.7}$$

where

$$\frac{1}{\alpha_1^+} = \frac{1}{\alpha_1} + \frac{h_1}{\lambda_1},\ \frac{1}{\alpha_2^+} = \frac{1}{\alpha_1^+} + \frac{h_2}{\lambda_2},\ \frac{1}{\alpha_3^+} = \frac{1}{\alpha_2^+} + \frac{h_3}{\lambda_3},$$

$$\frac{1}{\alpha_2^-} = \frac{1}{\alpha_3} + \frac{h_3}{\lambda_3},\ \frac{1}{\alpha_1^-} = \frac{1}{\alpha_2^-} + \frac{h_2}{\lambda_2},\ \frac{1}{\alpha_0^-} = \frac{1}{\alpha_1^-} + \frac{h_1}{\lambda_1},$$

$$Q_0 = R_0^+ + \alpha_0^- Q_1^-,\ Q_1 = Q_1^-\alpha_1^- + Q_0^+\alpha_1^+,\ Q_2 = Q_2^-\alpha_2^- + Q_1^+\alpha_2^+,$$

$$Q_3 = Q_3^-\alpha_3^- + Q_2^+\alpha_3^+,\ Q_1^+ = \frac{R_1^- + R_1^+}{\alpha_1^+} + Q_0^+,\ Q_2^+ = \frac{R_2^- + R_2^+}{\alpha_2^+} + Q_1^+,$$

$$Q_0^+ = \frac{R_0^+}{\alpha_1},\ Q_3^- = \frac{R_3^-}{\alpha_3},\ Q_2^- = \frac{R_2^- + R_2^+}{\alpha_2^-} + Q_3^-,\ Q_1^- = \frac{R_1^- + R_1^+}{\alpha_1^-} + Q_2^-.$$

In the case of the Dirichlet boundary condition ($\alpha_1; \alpha_3 \to \infty$) it follows from (4.7), that $u_0 = \theta_0$, $u_3 = \theta_3$.

If $\alpha_1 = 0$, then

$$Q_0^+\alpha_1^+ = R_0^+,\quad Q_1^+\alpha_2^+ = R_0^+ + R_1^+ + R_1^-,$$
$$Q_2^+\alpha_3^+ = R_0^+ + R_1^+ + R_1^- + R_2^+ + R_2^-,$$
$$\alpha_1^+ = \alpha_2^+ = \alpha_3^+ = 0.$$

If in addition $R_k = 0$, $k = \overline{0,3}$, then $u_0 = u_1 = u_2 = u_3 = \theta_3$.

5. APPROXIMATION OF INTEGRALS WITH THE HELP OF QUADRATURE FORMULAS

In a non-stationary case integrals $\tilde{R}_k^+, \tilde{R}_k^-$ are computed approximately with the help of quadrature formulas.

By means of substitutions $\xi_k = \dfrac{z - z_k}{h_k}$ or $\xi_k = \dfrac{z_{k+1} - z}{h_{k+1}}$ we find

$$\tilde{R}_k^- = c_k\rho_k h_k \int_0^1 \xi_k g_k^-(\xi)d\xi, \tag{5.1}$$
$$\tilde{R}_k^+ = c_{k+1}\rho_{k+1}h_{k+1} \int_0^1 \xi_k g_k^+(\xi)d\xi,$$

where $g_k^- = \dot{u}_k(\xi_k h_k + z_{k-1}, t)$, $g_k^+ = \dot{u}_{k+1}(z_{k+1} - \xi_k h_{k+1}, t)$.

Let us consider the approximation of the integral $I = \int_0^1 \xi g(\xi) d\xi$ by using the interpolation quadrature formulas

$$I = A_0 g(0) + A_1 g(1) + 0.5 C_0 g''(\tilde{\xi}),\ \tilde{\xi} \in (0,1),$$

where A_0, A_1, C_0 are undetermined coefficients.

Choosing functions g in the form $g(\xi) = \xi^i,\ i = 0,1,2$, we obtain the system of three equations

$$\frac{1}{2} = A_0 + A_1, \quad \frac{1}{3} = A_1, \quad \frac{1}{4} = A_1 + C_0.$$

It is easy to prove that $A_0 = \frac{1}{6},\ A_1 = \frac{1}{3},\ C_0 = -\frac{1}{12}$ is the solution of the system.

Thus we find

$$\begin{cases} \tilde{R}_k^- = c_k \rho_k h_k \Big(\frac{1}{6} \dot{u}_k(z_{k-1}, t) + \frac{1}{3} \dot{u}_k(z_k, t) - \frac{h_k^2}{24} \frac{\partial^2 \dot{u}_k(\eta_k^-, t)}{\partial z^2} \Big), \\ \tilde{R}_k^+ = c_{k+1} \rho_{k+1} h_{k+1} \Big(\frac{1}{3} \dot{u}_{k+1}(z_k, t) + \frac{1}{6} \dot{u}_{k+1}(z_{k+1}, t) \\ \quad - \frac{h_{k+1}^2}{24} \frac{\partial^2 \dot{u}_k(\eta_k^+, t)}{\partial z^2} \Big), \end{cases} \tag{5.2}$$

where $\eta_k^- \in (z_{k-1}, z_k),\ \eta_k^+ \in (z_k, z_{k+1})$.

After omitting the residual members in (5.2), we find the system of $N+1$ ODEs of the first order

$$\begin{cases} c_1 \rho_1 h_1 \Big(\frac{1}{3} \dot{u}_0(t) + \frac{1}{6} \dot{u}_1(t) \Big) = \frac{\lambda_1}{h_1} \Big(u_1(t) - u_0(t) \Big) - \alpha_1 \Big(u_0(t) - \theta_0 \Big) - I_0^+, \\ c_k \rho_k h_k \Big(\frac{1}{6} \dot{u}_{k-1}(t) + \frac{1}{3} \dot{u}_k(t) \Big) + c_{k+1} \rho_{k+1} h_{k+1} \Big(\frac{1}{3} \dot{u}_k(t) + \frac{1}{6} \dot{u}_{k+1}(t) \Big) \\ = \frac{\lambda_{k+1}}{h_{k+1}} \Big(u_{k+1}(t) - u_k(t) \Big) - \frac{\lambda_k}{h_k} \Big(u_k(t) - u_{k-1}(t) \Big) - I_k^- - I_k^+,\ k = \overline{1, N-1}, \\ c_N \rho_N h_N \Big(\frac{1}{6} \dot{u}_{N-1}(t) + \frac{1}{3} \dot{u}_N(t) \Big) \\ = \frac{\lambda_N}{h_N} \Big(u_N(t) - u_{N-1}(t) \Big) - \alpha_N \Big(u_N(t) - \theta_N \Big) - I_N^-, \end{cases} \tag{5.3}$$

where $\dot{u}_{k-1}(t) \equiv \dot{u}_k(z_{k-1}, t), \quad \dot{u}_k(t) \equiv \dot{u}_k(z_k, t)$. Here the continuity conditions of functions $u_k(z,t), \dot{u}_k(z,t)$ on joints of layers are used.

We find the distribution of the initial temperature at $t = 0$ in the form

$$u^{(0)}(z) = Bz + C,$$

coordinating it with the boundary conditions (1.3). Thus we obtain the

Table 1.
Dependence of temperatures on time:

a)

t	$u_0(t)$	$u_1(t)$	$u_2(t)$	$u_3(t)$
0.0	275.87	276.41	263.17	254.71
0.2	276.70	265.71	264. 86	253.91
0.4	276.82	265.40	265.17	253.82
0.6	276.82	265.34	265.22	253.86
0.8	276.79	265.33	265.23	253.92
1.0	276.76	265.32	265.23	253.98

b)

t	$u_0(t)$	$u_1(t)$	$u_2(t)$	$u_3(t)$
0.0	275.87	267.41	263.17	254.71
0.2	276.56	266.00	265.14	253.77
0.4	276.55	265.96	265.73	253.56
0.6	276.42	266.18	266.06	253.48
0.8	276.27	266.44	266.34	253.44
1.0	276.20	266.58	266.48	253.42

system of two algebraic equations for the determination of two constants B, C

$$B = \frac{\alpha_1 \alpha_N (\theta_N - \theta_0)}{\alpha_1(\lambda_N + \alpha_N z_N) + \alpha_N \lambda_N}, \quad C = \theta_0 + \frac{\lambda_1}{\alpha_1} B.$$

Then initial conditions of the system (5.3) can be presented in the form

$$u_k(0) = u^{(0)}(z), \quad k = \overline{0, N}.$$

6. NUMERICAL RESULTS

Let us consider numerical experiments in the case $q_k \equiv 0$, $N = 3$. We use the following values of plate parameters (in the wall of a house consisting of 3 layers: brick, metal, brick): $h_1 = h_3 = 0.4m$, $h_2 = 0.2m$ are the thickness of layers, $\rho_1 = \rho_3 = 1600\frac{kg}{m^3}$, $\rho_2 = 7800\frac{kg}{m^3}$ - the density of layers, $c_1 = c_3 = 750\frac{J}{kg.K}$, $c_2 = 500\frac{J}{kg.K}$ - specific thermal capacities of layers, $\lambda_1 = \lambda_3 = 0.8\frac{W}{m.K}$, $\lambda_2 = 59\frac{W}{m.K}$ - factors of heat conductivity of layers, $\alpha_1 = 1\frac{W}{m^2.K}$, $\alpha_3 = 10\frac{W}{m^2.K}$ - the convective heat transfer coefficients and $\theta_0 = 293K$, $\theta_3 = 253K$- the air temperatures, respectively, on the bottom and top of the plate in Kelvin degree.

Calculations and their graphic visualization were made by means of mathematical system MAPLE - 5 RELEASE 4. From Table 1a) we can see the dependence of temperature on time.

We also solved the inverse problem. From (4.7) the convective heat transfer coefficients α_1, α_3 and temperature on a joint of layers u_1, u_2 were found, when air temperatures around a plate θ_0, θ_3 and on boundary of a plate u_0, u_3 were known (see Table 2). The results for the case $q_1 = 0$, $q_2 = 2000\ \frac{W}{m^3}$, $q_3 = 0$ are presented in Table 1b).

7. THE CONCLUSION

1. The proposed method allows us to reduce 1D heat transfer problem to the system of the ordinary differential equations (5.3).

Table 2.
The results of the inverse problems in the stationary case.

θ_0	θ_3	u_0	u_3	u_1	u_2	α_1	α_3
293	300	296	299	297.99	298.01	1.04	11.43
293	253	274	255	264.43	264.37	1.00	10.09
293	310	296	300	297.99	298.01	1.34	0.40
293	243	274	255	264.53	264.47	1.01	1.60
293	297	295	296	295.50	295.50	0.50	1.01
293	298	295	297	296.00	296.00	1.01	2.02
293	300	295	299	297.00	297.00	2.02	4.04
293	302	296	300	298.00	298.00	1.35	2.02

2. The method allows us to find the distribution of temperature both on a joint of layers and in any place of a horizontal plate.
3. By means of the stationary solution formulas (4.5), (4.6), it is possible to solve an inverse problem, i.e., to determine α_1, α_N for known temperatures on the bottom and top surface of a plate $u_0,\ u_N$ and external temperatures $\theta_0,\ \theta_N$, i.e., to solve the system of $N+1$ algebraic equations (4.5), (4.6), concerning unknown values $\alpha_1,\ \alpha_N,\ u_k,\ k=\overline{1, N-1}$.

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Paprastieji inžinerinio skaičiavimo metodai šilumos laidumo uždaviniams spręsti

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Darbe nagrinėjami du – baigtinių skirtumų ir kraštinių elementų – metodai šilumos pasiskirstymo daugiasluoksnėje aplinkoje uždaviniams spręsti. Šiais metodais dviejų kintamųjų uždavinys dalinėmis išvestinėmis pakeičiamas pradiniu – kraštiniu paprastųjų diferencialinių lygčių sistemos uždaviniu. Tokia procedūra suteikia galimybę gauti paprastus inžinerinius algoritmus, skirtus spręsti šilumos laidumo lygtį daugiasluoksnėje srityje. Stacionariu atveju įmanoma nustatyti tikslų skirtumų schemos sprendinį. Darbe nagrinėtas atvirkščias uždavinys. Skaitinio eksperimento metu gauti šilumos laidumo koeficientai ir temperatūros vidiniuose sluoksniuose priklausomai nuo išorinių plokštės duomenų.